

## A NEW RANKING OF INTUITIONISTIC FUZZY NUMBERS WITH DISTANCE METHOD BASED ON THE CIRCUMCENTER OF CENTROIDS

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### ABSTRACT

In this paper, a new ranking method for intuitionistic fuzzy numbers is proposed based on the circumcenter of centroids of membership function and non-membership function of intuitionistic fuzzy numbers.

**KEYWORDS:** Circumcenter of Centroids of Trapezoidal Intuitionistic Fuzzy Number, Fuzzy Number, Trapezoidal Fuzzy Number, Trapezoidal Intuitionistic Fuzzy Number

### 1. INTRODUCTION

Zadeh [1] introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life. The concept of Intuitionistic fuzzy set[4,6] can be viewed as an appropriate/alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. The intuitionistic fuzzy sets were first introduced by Atanassov[4] which is a generalization of the concept of fuzzy set[1]. Ranking fuzzy numbers is one of the fundamental problems of fuzzy arithmetic and fuzzy decision making. Fuzzy numbers must be ranked before an action is taken by a decision maker. Real numbers can be linearly ordered by the relation  $\leq$  or  $\geq$ , however this type of inequality does not exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than other. An efficient method for ordering the fuzzy numbers is by the use of a ranking function, which maps each fuzzy number into the real line, where a natural order exists. The concept of ranking function for comparing normal fuzzy numbers is compared in Jain[2]. Abbasbandy, Hajjari[12] presented a new approach for ranking of trapezoidal fuzzy numbers. Wang. and Kerre[8] presented properties for the ordering of fuzzy quantities. In Mitchell[10] and Nayagam[11] some methods for ranking of intuitionistic fuzzy numbers were introduced. Grzegorzewski [9] suggested the method of ranking Intuitionistic fuzzy numbers and an ordering method for Intuitionistic fuzzy numbers by using the expected interval of an Intuitionistic fuzzy number. Based on the characteristic value for a fuzzy number introduced in Kuo-Ping Chiao [7], an ordering method for Intuitionistic fuzzy number is proposed by Hassan Mishmast Nehi [14]. Phani Bushan Rao and Ravi Shankar[15] presented a method for ranking fuzzy numbers using Circumcenter of Centroids and an index of modality.

In this paper, a new method for ranking intuitionistic fuzzy numbers is proposed based on the circumcenter of centroids of membership function and non-membership function of intuitionistic fuzzy numbers. In a trapezoidal intuitionistic fuzzy number, first the trapezoids of membership function and nonmembership function are divided into three plane figures a triangle, a rectangle and a triangle respectively. Then the centroids of three plane figures of membership function and nonmembership function are calculated followed by the calculation of the circumcenter of these centroids. Finally ranking functions of membership function and nonmembership function of intuitionistic fuzzy number are defined which are the Euclidean distances between the circumcenter point and the original point respectively to rank intuitionistic fuzzy numbers.

## 2. PRELIMINARIES ([3], [4], [5], [6], [7], [13], [14])

**Definition 2.1:** If  $X$  is a collection of objects denoted generically by  $x$ , then a fuzzy set  $A$  in  $X$  is defined to be a set of ordered pairs  $A = \{(x, \mu_A(x)) / x \in X\}$ , where  $\mu_A(x)$  is called the membership function for the fuzzy set. The membership function maps each element of  $X$  to a membership value between 0 and 1.

**Remark:** We assume that  $X$  is the real line  $R$

**Definition 2.2:** The support of a fuzzy set  $A$  is the set of points  $x$  in  $X$  with  $\mu_A(x) > 0$ .

**Definition 2.3:** The core of a fuzzy set  $A$  is the set of points  $x$  in  $X$  with  $\mu_A(x) = 1$ .

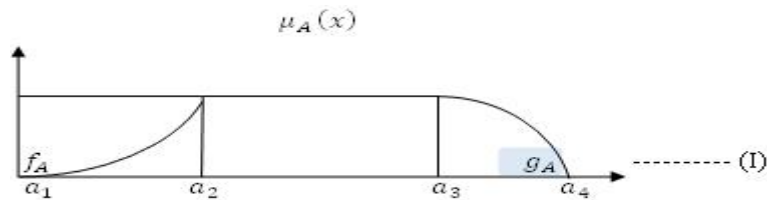
**Definition 2.4:** A fuzzy set  $A$  of the real line  $R$  with the membership function  $\mu_A : R \longrightarrow [0,1]$  is called a fuzzy number if

- $A$  is normal . ie. There exist an element  $x_0$  such that  $\mu(x_0) = 1$
- $A$  is fuzzy convex.  
ie.  $\forall x_1, x_2 \in R, \forall \lambda \in [0,1], \mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2)$
- $\mu_A$  is upper semi continuous.
- $A$  is bounded.

It is known that for any fuzzy number  $A$ , there exists four numbers  $a_1, a_2, a_3, a_4 \in R$  and two functions  $f_A, g_A : R \longrightarrow [0,1]$  where  $f_A$  is non decreasing and  $g_A$  is nonincreasing, such that we can describe a membership function  $\mu_A$  in a following manner

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \leq a_1 \\ f_A(x) & \text{if } a_1 < x < a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ g_A(x) & \text{if } a_3 < x < a_4 \\ 0 & \text{if } a_4 \leq x \end{cases} \quad (1)$$

functions  $f_A$  and  $g_A$  are called the left side and the right side of a fuzzy number  $A$  respectively.



**Figure 1: Fuzzy Number**

**Definition 2.5:** The  $\alpha$  – cut of a fuzzy number  $A$  is a non fuzzy set defined as

$$A_\alpha = \{x \in R / \mu_A(x) \geq \alpha\}$$

Every  $\alpha$  – cut of a fuzzy number  $A$  is a closed interval. Hence we have  $A_\alpha = [A_L(\alpha), A_U(\alpha)]$

Where  $A_L(\alpha) = \inf\{x \in R / \mu_A(x) \geq \alpha\}$  and  $A_U(\alpha) = \sup\{x \in R / \mu_A(x) \geq \alpha\}$

**Definition 2.6:** Let  $X$  be the universal set. An Intuitionistic fuzzy set (IFS)  $A$  in  $X$  is given by

$$A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\} \text{ where the functions } \mu_A, \nu_A : X \longrightarrow [0, 1] \text{ are functions such that } 0 \leq \mu(x) + \nu(x) \leq 1 \forall x \in X.$$

For each  $x$  the numbers  $\mu_A(x)$  and  $\nu_A(x)$  represent the degree of membership and degree of non- membership of the element  $x \in X$  to the set  $A$ , which is a subset of  $X$ , respectively.

**Definition 2.7:** For each IFS  $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$  in  $X$ , we will call

$$\Pi_A = 1 - \mu(x) - \nu(x) \text{ as the Intuitionistic fuzzy index of } X \text{ in } A. \text{ It is obvious that } 0 \leq \Pi_A \leq 1 \forall x \in X.$$

**Definition 2.8:** The two kinds of  $\alpha$  – cut for Intuitionistic fuzzy sets is defined as

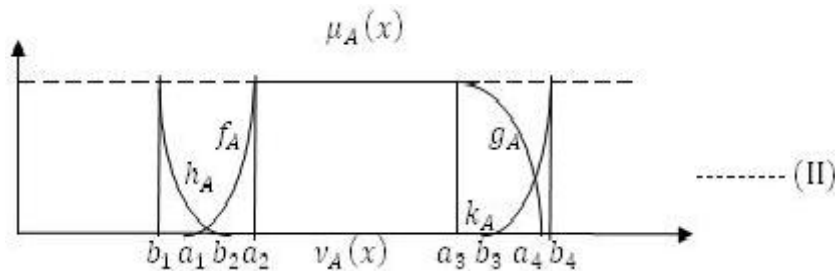
$$A_\alpha = \{x \in R / \mu_A(x) \geq \alpha\}, \quad A_\alpha = \{x \in R / \nu_A(x) \geq \alpha\}$$

**Definition 2.9:** An IFS  $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$  is called IF- normal, if there exists at least two points  $x_0, x_1 \in X$  such that  $\mu_A(x_0) = 1, \nu_A(x_1) = 1$ .

**Definition 2.10:** An IFS  $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$  of the real line is called IF- convex if  $\forall x_1, x_2 \in R, \forall \lambda \in [0,1], \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2)$  and  $\nu_A(\lambda x_1 + (1-\lambda)x_2) \geq \nu_A(x_1) \wedge \nu_A(x_2)$ . Thus  $A$  is IF-convex if its membership function is fuzzy convex and its non membership function is fuzzy concave.

**Definition 2.11:** An IFS  $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$  of the real line is called an Intuitionistic fuzzy number (IFN) if

- $A$  is IF-normal,
- $A$  is IF-convex,
- $\mu_A$  is upper semi continuous and  $\nu_A$  is lower semi continuous.
- $A = \{x \in X / \nu_A(x) < 1\}$  is bounded.



**Figure 2: Intuitionistic Fuzzy Number**

For any IFN  $A$  there exists eight numbers  $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in R$  such that

$b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$  and four functions  $f_A, g_A, h_A, k_A : R \longrightarrow [0,1]$ , called the sides of a fuzzy number, where  $f_A$  and  $k_A$  are nondecreasing and  $g_A$  and  $h_A$  are nonincreasing, such that we can describe a membership function  $\mu_A$  and non membership  $\nu_A$  in the following form

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \leq a_1 \\ f_A(x) & \text{if } a_1 < x < a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ g_A(x) & \text{if } a_3 < x < a_4 \\ 0 & \text{if } a_4 \leq x \end{cases} \text{ and } v_A(x) = \begin{cases} 0 & \text{if } x \leq b_1 \\ h_A(x) & \text{if } b_1 < x < b_2 \\ 0 & \text{if } b_2 \leq x \leq b_3 \\ k_A(x) & \text{if } b_3 < x < b_4 \\ 0 & \text{if } b_4 \leq x \end{cases} \quad (2)$$

**Definition 2.12:** The  $\alpha$  – cuts of a Intuitionistic fuzzy number  $A$  are a non fuzzy sets defined as

$$(A^+)_{\alpha} = \{x \in R / \mu_A(x) \geq \alpha\} = A_{\alpha}, (A^-)_{\alpha} = \{x \in R / 1 - v_A(x) \geq \alpha\} \\ = \{x \in R / v_A(x) \leq 1 - \alpha\} = A^{1-\alpha}$$

Every  $\alpha$  – cut  $(A^+)_{\alpha}$  or  $(A^-)_{\alpha}$  is a closed interval.

Hence  $(A^+)_{\alpha} = [A_L^+(\alpha), A_U^+(\alpha)]$  and  $(A^-)_{\alpha} = [A_L^-(\alpha), A_U^-(\alpha)]$  respectively, where

$$A_L^+(\alpha) = \inf\{x \in R / \mu_A(x) \geq \alpha\}, A_U^+(\alpha) = \sup\{x \in R / \mu_A(x) \geq \alpha\}$$

$$A_L^-(\alpha) = \inf\{x \in R / v_A(x) \leq 1 - \alpha\}, A_U^-(\alpha) = \sup\{x \in R / v_A(x) \leq 1 - \alpha\}$$

If the sides of the fuzzy numbers  $A$  are strictly monotone then, the convention that

$$f_A^{-1}(\alpha) = A_L^+(\alpha), g_A^{-1}(\alpha) = A_U^+(\alpha), h_A^{-1}(\alpha) = A_L^-(\alpha), \text{ and } k_A^{-1}(\alpha) = A_U^-(\alpha)$$

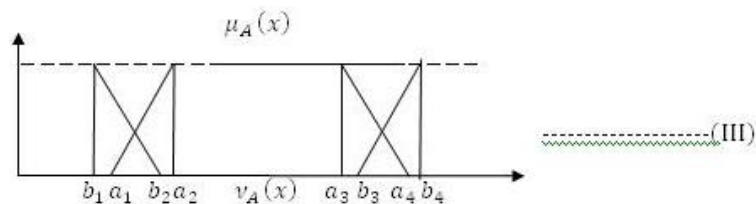
In particular if the decreasing functions  $g_A$  and  $h_A$  and increasing functions  $f_A$  and  $k_A$  be linear then we will have the trapezoidal intuitionistic fuzzy numbers (TIFN).

**Definition 2.13:**  $A$  is a trapezoidal intuitionistic fuzzy number with parameters

$$b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4 \text{ and denoted by } A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$$

In this case,

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < a_1 \\ \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ \frac{x-a_4}{a_3-a_4} & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{if } a_4 < x \end{cases} \text{ and } v_A(x) = \begin{cases} 0 & \text{if } x < b_1 \\ \frac{x-b_1}{b_2-b_1} & \text{if } b_1 \leq x \leq b_2 \\ 0 & \text{if } b_2 \leq x \leq b_3 \\ \frac{x-b_4}{b_3-b_4} & \text{if } b_3 \leq x \leq b_4 \\ 0 & \text{if } b_4 < x \end{cases} \quad (3)$$



**Figure 3: Trapezoidal Intuitionistic Fuzzy Number**

If in a TIFN  $A$ , we let  $b_2 = b_3$  ( and hence  $a_2 = a_3$ ) then we will give a Triangular Intuitionistic fuzzy number (TrIFN) with parameters  $b_1 \leq a_1 \leq b_2 (a_2 = a_3 = b_3) \leq a_4 \leq b_4$  and denoted by  $A = (b_1, a_1, b_2, a_4, b_4)$ . And in this paper,  $I$  is the set of trapezoidal intuitionistic fuzzy numbers.

### 3. PROPOSED RANKING METHOD FOR IFN

**Definition 2.14:** The centroid of a trapezoid is considered as the balancing point of the trapezoid. Divide the trapezoid of membership function of intuitionistic fuzzy number into three plane figures. These three plane figures are a triangle, a rectangle and again a triangle respectively. The circumcenter of the centroids of these three plane figures is taken as the point of reference to define the ranking of intuitionistic fuzzy numbers. The reason for selecting this point as a point of reference is that each centroid point ( $G_1$  of triangle,  $G_2$  of rectangle,  $G_3$  of triangle) are balancing points of each individual plane figure and the circumcenter of these centroid points is equidistant from each vertex (which are centroids). Therefore, this point would be a better reference point than the centroid point of the trapezoid.

Consider a Trapezoidal Intuitionistic fuzzy number  $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$  (Figure 4). The centroids of the three plane figures of membership function are  $G_1 = \left(\frac{(a_1+2a_2)}{3}, \frac{1}{3}\right)$ ,  $G_2 = \left(\frac{(a_2+a_3)}{2}, \frac{1}{2}\right)$  and  $G_3 = \left(\frac{(2a_3+a_4)}{3}, \frac{1}{3}\right)$  respectively. Equation of the line  $G_1G_3$  is  $y = \frac{1}{3}$  and  $G_2$  does not lie on the line  $G_1G_3$ . Therefore  $G_1, G_2$  and  $G_3$  are non-collinear and they form a triangle.

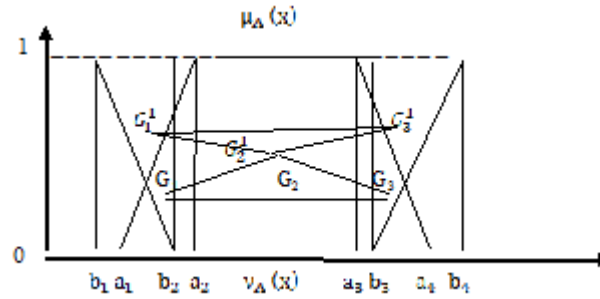


Figure 4: Circumcenter of Centroids

The Circumcenter  $C_{A_\mu}(\bar{x}_0, \bar{y}_0)$  of the triangle with vertices  $G_1, G_2$  and  $G_3$  of the membership function of the trapezoidal intuitionistic fuzzy number

$A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$  is defined as

$$C_{A_\mu}(\bar{x}_0, \bar{y}_0) = \left( \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}, \frac{(2a_1 + a_2 - 3a_3)(2a_4 + a_3 - 3a_2) + 5}{12} \right)$$

Also divide the trapezoid of nonmembership function of intuitionistic fuzzy number into three plane figures. These three plane figures are a triangle, a rectangle and again a triangle respectively. And the centroids of the three plane figures of nonmembership function are  $G_1^1 = \left(\frac{(b_1+2b_2)}{3}, \frac{2}{3}\right)$ ,  $G_2^1 = \left(\frac{(b_2+b_3)}{2}, \frac{1}{2}\right)$  and  $G_3^1 = \left(\frac{(2b_3+b_4)}{3}, \frac{2}{3}\right)$  respectively. Equation of the line  $G_1^1G_3^1$  is  $y = \frac{2}{3}$  and  $G_2^1$  does not lie on the line  $G_1^1G_3^1$ . Therefore  $G_1^1, G_2^1$  and  $G_3^1$  are non-collinear and they form a triangle. And the Circumcenter  $C_{A_\nu}(\bar{x}_1, \bar{y}_1)$  of the triangle with vertices  $G_1^1, G_2^1$  and  $G_3^1$  of the non-membership function of the trapezoidal intuitionistic fuzzy number  $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$  is defined as

$$C_{A_\nu}(\bar{x}_1, \bar{y}_1) = \left( \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}, \frac{(2b_1 + b_2 - 3b_3)(-2b_4 - b_3 + 3b_2) + 7}{12} \right)$$

As a special case, if in a TIFN  $A$ , we let  $b_2 = b_3$  (and hence  $a_2 = a_3$ ) then we will get a triangular intuitionistic

fuzzy number (TrIFN) with parameters  $b_1 \leq a_1 \leq b_2$  ( $a_2 = a_3 = b_3$ )  $\leq a_4 \leq b_4$  and denoted by  $A = (b_1, a_1, b_2, a_4, b_4)$ . The Circumcenters of centroids of the membership function and nonmembership function of the triangular intuitionistic fuzzy number respectively are defined as

$$C_{A_\mu}(\bar{x}_0, \bar{y}_0) = \left( \frac{a_1 + 4a_2 + a_4}{6}, \frac{(a_1 - a_2)(a_4 - a_2) + 5}{3} \right) \text{ and}$$

$$C_{A_\gamma}(\bar{x}_1, \bar{y}_1) = \left( \frac{b_1 + 4b_2 + b_4}{6}, \frac{(b_1 - b_2)(-b_4 + b_2) + 7}{3} \right)$$

**Definition 2.15:** The ranking function of the trapezoidal intuitionistic fuzzy number  $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$  for membership function and non-member ship function are defined as  $R(A_\mu) = \sqrt{\bar{x}_0^2 + \bar{y}_0^2}$  and  $R(A_\gamma) = \sqrt{\bar{x}_1^2 + \bar{y}_1^2}$  which are the Euclidean distances from the Circumcenters of the Centroids as defined in Definition 2.14 and the original points respectively.

Using the above definitions we compare two trapezoidal intuitionistic fuzzy numbers  $A$  and  $B$  using the following steps:

**Step 1:** Compare  $R(A_\mu)$  and  $R(B_\mu)$ . If they are equal, then go to the step 2. Otherwise rank  $A$  and  $B$  according to the relative position of  $R(A_\mu)$  and  $R(B_\mu)$ .

**Step 2:** Compare  $R(A_\gamma)$  and  $R(B_\gamma)$ . If they are equal, then  $A$  and  $B$  are equal. Otherwise rank  $A$  and  $B$  according to the relative position of  $-R(A_\gamma)$  and  $-R(B_\gamma)$ .

Therefore for any two trapezoidal intuitionistic fuzzy numbers  $A$  and  $B \in I$ , we define the ranking of  $A$  and  $B$  as follows:

- $R(A_\mu) > R(B_\mu)$  if and only if  $A < B$ ,
- $R(A_\mu) < R(B_\mu)$  if and only if  $A \prec B$ ,
- $R(A_\mu) = R(B_\mu)$  and  $R(A_\gamma) = R(B_\gamma)$  if and only if  $A \sim B$ ,
- $R(A_\mu) = R(B_\mu)$  and  $-R(A_\gamma) > -R(B_\gamma)$  if and only if  $A < B$ .
- $R(A_\mu) = R(B_\mu)$  and  $-R(A_\gamma) < -R(B_\gamma)$  if and only if  $A < B$ .

Then the order  $\leq$  and  $\prec$  is formulated as  $A \leq B$  if and only if  $A \succ B$  or  $A \sim B$ ,  $A \prec B$  if and only if  $A < B$  or  $A \sim B$ .

#### 4. CONCLUSIONS

In this paper, we have found the circumcenter of centroids of membership function and non-membership function of a intuitionistic fuzzy numbers and proposed a distance method for ranking of Intuitionistic fuzzy numbers based on the circumcenter of centroids. The proposed method provides the exact ordering of intuitionistic fuzzy numbers. This approach can be applied to rank the intuitionistic fuzzy numbers in solving different fuzzy optimization problems.

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